# Discussion on Limit Equilibrium Analysis Models and Modes for Reinforced Soil Slopes and Walls


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## *Abstract*

*Reinforced soil structures, retaining walls and steep slopes, are normally designed by limit equilibrium methods. In this case, three "independent" analyses are commonly performed - internal, external and local stability analysis - in order to define a typical cross section for the structure.*

*Beside this failure modi the so-called "compound" mode can occur: the failure surface crosses both the reinforced fill and the unreinforced backfill. Unfortunately, this mode is being often forgotten by design engineers and owners, which can lead to failure. Comparative calculations are presented using common geotechnical limit-equilibrium-based procedures, showing the risks of inappropriate "cheap" design "forgetting" the "compound" mode. Additionally, some other "risky" issues are discussed.*


## 1 INTRODUCTION

Reinforced soil structures (retaining walls and steep slopes) are normally designed by limit equilibrium methods. Three "independent" analyses are commonly performed - internal, external and facing stability in order to define a typical cross section for the structure including geometry, design strength, length and spacing of the reinforcement.

The internal stability analysis (Figure 1a) includes failure surfaces (of any type) crossing only the reinforced zone, but neither the soil behind ("backfill") nor the soil below ("foundation soil"). The external stability analysis (Figure 1b) includes failure surfaces (of any type) running through the backfill and the foundation soil without crossing the reinforced zone, which is supposed to be a stable quasi-homogeneous block in that case. The facing stability analysis deals only with the locally limited zone of the facing of the structure and is facing-specific.

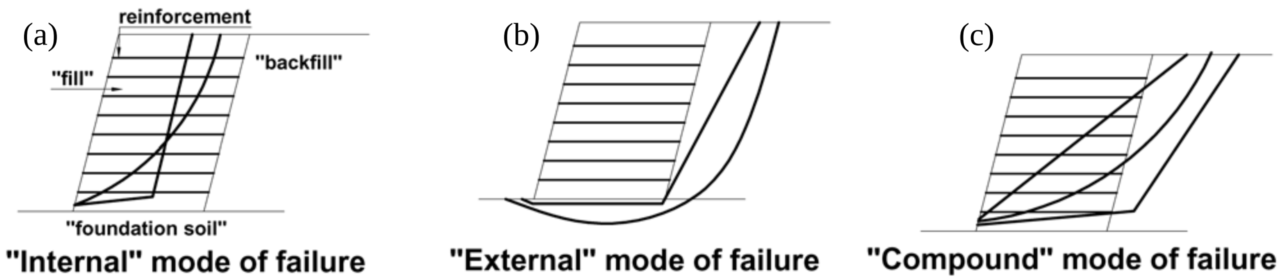


Figure 1 Overview of possible failure modes

In many codes, recommendations etc. and especially very often in company-specific "freeware" no further possible modes of failure are being analyzed or checked. The consequences could be dramatic based on the authors experience and some cases reported in the literature. Such a typical "dangerous" mode is the so-called "compound" mode: a failure surface (of any type) crosses both the reinforced zone and the backfill (Figure1c). Unfortunately, relatively often this mode remains neglected by geotechnical consultants and project engineers. We will focus on the risks of checking only “internal” and “external” stability modes without the “compound” mode showing the results of comparative stability analyses without and with the “compound” failure mode for some typical reinforced slope and wall cases.

## 2 LIMIT EQUILIBRIUM DESIGN METHODS: BASIC CONCEPTS

Several limit equilibrium analysis and design methods are available using different failure surfaces and assumptions (e.g. Rankine, Coulomb, Bishop, Janbu etc.). Many corresponding software tools are available in the market (some of them correct, some of them not really) even as “freeware” from companies, and are widely used by geotechnical engineers.

Generally, limit equilibrium concepts are simplified tools for design; each proposed method for analysis has its own simplifications and consequent limitations.

Nevertheless, structures being designed based on calculations using specific methods, which are not valid for the corresponding case, are not a rare practice. For saving efforts or due to incompetence, limitations of the method are simply forgotten in many cases and simplified methods are applied where they couldn’t be. In general, one important hypothesis is the shape and geometry of failure surface that is considered by each method as being the most critical one.

Silva and Vidal (1999) show, through parametric analysis, the magnitude of the possible error by using both Rankine and Coulomb hypothesis for designing reinforced retaining walls for situations outside of the frame of the respective hypotheses. These are maybe the mostly used limit equilibrium methods for such a purpose, mainly because of their simplicity. They both consider linear failure surface, but are strictly limited to vertical (or nearly vertical) face structures. Also, soil type, general geometry of the structure, and load conditions are important parameters for defining the validity of these methods. Table 1 presents some results and comparisons of these two methods with the so-called Two-Part-Wedge Method, which is more universal. The results are in our opinion alarming.

Table 1 Comparisons between distinct analyses proposals (Silva and Vidal, 1999).

| $\beta$ (°) | c (kPa) | $\phi$ (°) | Coulomb | | Rankine | | Two Part Wedge | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | F (kN/m) | $\theta$ (°) | F (kN/m) | $\theta$ (°) | F (kN/m) | $\theta_1$ (°) | $\theta_2$ (°) | d (m) |
| 85 | 0 | 30 | 98 | 56 | 103 | 60 | 98 | 56 | - | - |
| | 10 | 19 | 68 | 51 | - | - | 69 | 47 | 52 | 0,5 |
| | 0 | 19 | 154 | 50 | 163 | 54 | 154 | 50 | - | - |
| 80 | 0 | 30 | 84 | 53 | 91 | 60 | 84 | 52 | 54 | 1,0 |
| | 10 | 19 | 50 | 49 | - | - | 55 | 36 | 52 | 0,9 |
| | 0 | 19 | 138 | 46 | 152 | 54 | 138 | 44 | 48 | 1,0 |
| 75 | 0 | 30 | 71 | 51 | 80 | 60 | 73 | 46 | 54 | 1,4 |
| | 10 | 19 | 33 | 46 | - | - | 44 | 28 | 52 | 1,2 |
| | 0 | 19 | 122 | 43 | 141 | 54 | 126 | 35 | 47 | 1,4 |
| 70 | 0 | 30 | 60 | 48 | 68 | 60 | 63 | 41 | 54 | 1,7 |
| | 10 | 19 | 17 | 44 | - | - | 33 | 23 | 51 | 1,5 |
| | 0 | 19 | 109 | 40 | 129 | 54 | 116 | 28 | 47 | 1,7 |

Boundary conditions:

- Wall Height: 6m
- No surcharge
- No backslope
- $\gamma_{embankment}$: 19kN/m$^3$
- Friction between slices: $\phi_w=\phi$
- No factor of safety applied

Legend:

- $\beta$: face inclination to the horizontal
- c, $\phi$: resistance parameters of the embankment soil
- $\theta$: rupture surface inclination
- $\theta_1$, $\theta_2$: inclination of the two parts of the two-part-wedge failure surface
- d: distance from the embankment toe to the failure surface deflection point (two part wedge method)
- F: total horizontal driving force (value for reinforcement design)

From the results presented in Table 1, it is clear that linear one-part failure surface is not a realistic hypothesis for structures with non-vertical facing, despite the traditional way of considering $\beta \geq 70°$ as being the condition for the application of such methods. In the case of non vertical structures, a two-part-wedge tends to be formed. This tendency is even more clear in the case of cohesive soils. Also, it is remarkable that Rankine is unsafe in the case of non vertical structures, and even not valid in the case of cohesive soils.

Besides the geometry and type of the failure surface, it is clear that the possible zone of the structure crossed by this surface (in other words: the searching area for critical surfaces), is important. Generally, one should first look for all failure surfaces possible from the point of view of soil mechanics, and then apply appropriate methods to analyze the situations.

## 3 STEEP REINFORCED SLOPES AND WALLS: "COMPOUND" MODE

Some philosophical points: "Internal" and "external" modes are only simplifications for analytical purposes. Real structure's behavior doesn't make any distinction between reinforced zone, backslope, retained soil and foundation layers. Nature simply searches for the limit equilibrium state of lowest stability level and finds the most unstable surface, independently of its path and its geometry.

In many cases a smooth (e.g. circular, log-spiral) or polygonal failure surface will cross both the reinforced and unreinforced zones, resulting from the point of view of stability analysis in the "compound" mode of failure mentioned in Chapter 1.

As mentioned above, the "compound" mode is very often not being checked. In our experience the main reasons for such an omission are:

(a) insufficient competence in soil mechanics;
(b) absence of explicit obligation to check the "compound" mode in some design codes and recommendations;
(c) saving time and money at the stage of design analyses;
(d) really good software is not available with the design engineer;
(e) creation of a "cheap" design "saving" reinforcement lengths at the expense of risk of possible compound failure mode.

Some comments to the reasons above:

- reason (a) – no help;
- reason (b) – codes and recommendations include often only the minimum requirements on design procedures; one has to think;
- reason (c) – these are savings not at the right place; one should better perform a really optimized design;
- reason (d) – powerful software is available in the market (e.g. GGU-Stability V. 6.16 or ReSlope V. 3.0); "freeware" e.g. from reinforcement suppliers should be used very carefully; it is often not really complete;
- reason (e) – the only help is the design to be re-checked by a competent person using appropriate state-of-the-art methods and software.

The fact that geotechnical software becomes more and more user-friendly can not substitute competence and thinking.

The authors have performed comparative stability calculations for some typical cases, checking all failure modes incl. the "compound" one.

## 4 COMPARATIVE STABILITY CALCULATIONS

A large number of stability analyses of typical cross sections of geogrid-reinforced slopes and retaining walls were performed. A German software for stability analysis of geotechnical structures (GGU-Stability V. 6.16) was used in that case. Limit equilibrium analyses were performed by two different methods: Bishop and Janbu, according to the German Standard for stability analysis (DIN 4084).

Three different cases were simulated. Internal, external and compound modes were analyzed. They are described in the following chapters and the results of each case are summarized in the Tables 1, 2 and 3, respectively. To allow for an easy comparison in terms of reinforcement quantity for all cases reinforcement layers with the same design strength for the entire height were assumed, which is not really an optimized design. For all cases typical fill, backfill and foundation soils were chosen:

- fill: $\varphi$ = 32.5°; c = 0 kN/m²; $\gamma$ = 20 kN/m³
- backfill: $\varphi$ = 30.0°; c = 0 kN/m²; $\gamma$ = 19 kN/m³
- foundation soil: $\varphi$ = 25.0°; c = 5 kN/m²; $\gamma$ = 18 kN/m³.

4.2 First case: reinforced steep slope (Fig. 2, Table 2)

Input:

- Height: 6 m
- Facing: ~ 65° (~ 2V:1H)
- Backslope (4 m high): 1V:3H
- No surcharge
- Required factor of safety: FOS ≥ 1.4 (DIN 4084)
- For all 13 geogrid layers $F_{design}$ = const = 15 kN/m
- Bond coefficient: for pull-out = 0.8, for shear = 1.0 (geogrids Fortrac®)
- All vertical spacings: 0.5 m
- Typical fill, backfill and foundation soil (see above)

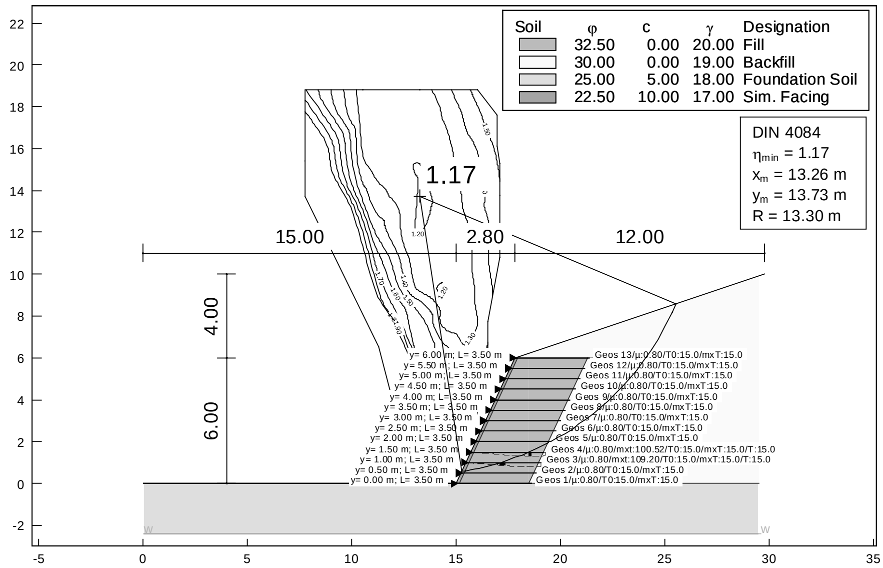


Figure 2a Reinforced slope, short reinforcement, compound mode, Bishop

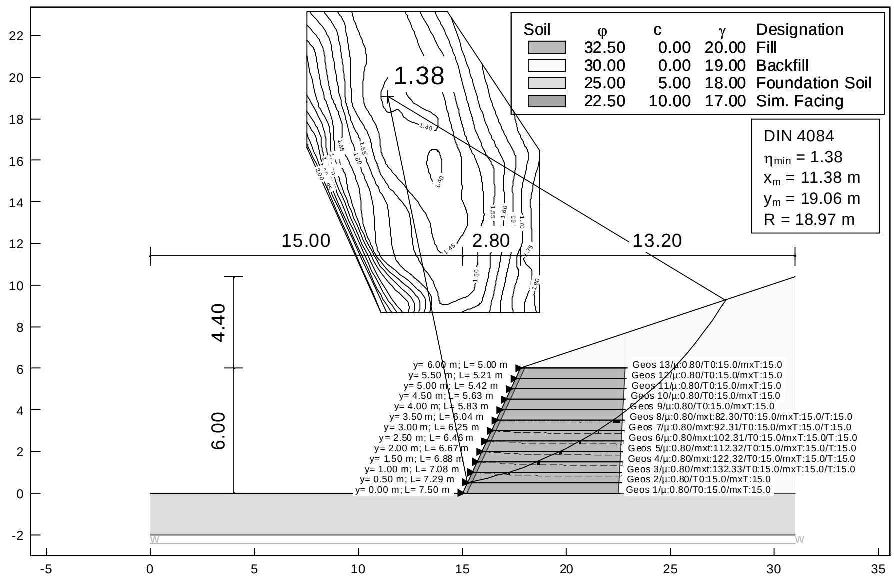


Figure 2b Reinforced steep slope, long reinforcement, compound mode, Bishop

Table 2 First case analysis results (FOS):

| | INTERNAL MODE | | EXTERNAL MODE | | COMPOUND MODE | |
|---|---|---|---|---|---|---|
| | L=3.5m [1)] | L=6.25m [1)] | L=3.5m[1)] | L=6.25m [1)] | L=3.5m[1)] | L=6.25m [1)] |
| BISHOP | 1.42 | 1.46 | 1.21 | 1.42 | 1.17 | 1.38 |
| JANBU | 1.56 | 1.71 | 0,92 | 1.34 | 1.04 | 1.32 |

1) L: average reinforcement length

Through internal stability analysis only, for a predefined reinforcement configuration (spacing and number of layers) and strength, geogrid layers with 3.5 m length would satisfy the required FOS. But, performing a complete analysis (internal + external + compound), the resulting FOS is below the minimum required. Geogrid layers with an average length of at least 6.25 m should be considered in this case. Because design strength of geogrids and vertical spacing are the same for all analyses, almost 80% more reinforcement, due to the increased length required, would be necessary to cover all failure modes with a more or less sufficient FOS.

Note: the “compound” mode controls the design (lowest FOS for the more or less sufficient length of L = 6.25 m)!

4.2 Second case: reinforced vertical wall (Fig. 3, Table 3)

Input:

- Height: 6 m
- Facing: 90° (vertical face)
- No backslope
- No surcharge
- Required factor of safety: FOS ≥ 1.4 (DIN 4084)
- For all 12 geogrid layers $F_{design}$ = const = 25 kN/m
- Bond coefficient: for pull-out = 0.8, for shear = 1.0 (geogrids Fortrac®)
- All vertical spacings: 0.5 m
- Typical fill, backfill and foundation soil (see above)

Table 3 Second case analysis results (FOS):

| | INTERNAL MODE | | EXTERNAL MODE | | COMPOUND MODE | |
|---|---|---|---|---|---|---|
| | L=2.5m[1)] | L=4.0m[1)] | L=2.5m[1)] | L=4.0[1)]m | L=2.5m[1)] | L=4.0m[1)] |
| BISHOP | 1.45 | 1.42 | 1.29 | 1.49 | 1.22 | 1.53 |
| JANBU | 1.80 | 1.63 | 1.18 | 1.46 | 1.10 | 1.35 |

1) L: average reinforcement length

Similarly to the First case, through internal stability analysis only, 2.5 m long geogrid layers would satisfy the required FOS. The complete analysis results in L ≥ 6.25 m. Note: the ”compound” mode controls the design again. The complete correct design requires 60% more reinforcement than the internal stability alone.

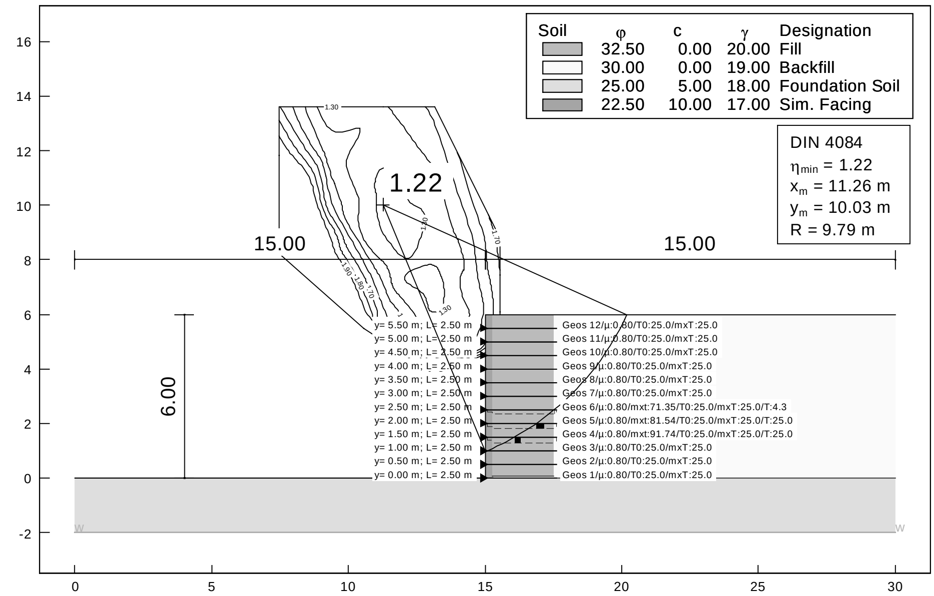


Figure 3a Reinforced vertical wall, short reinforcement, compound mode, Bishop

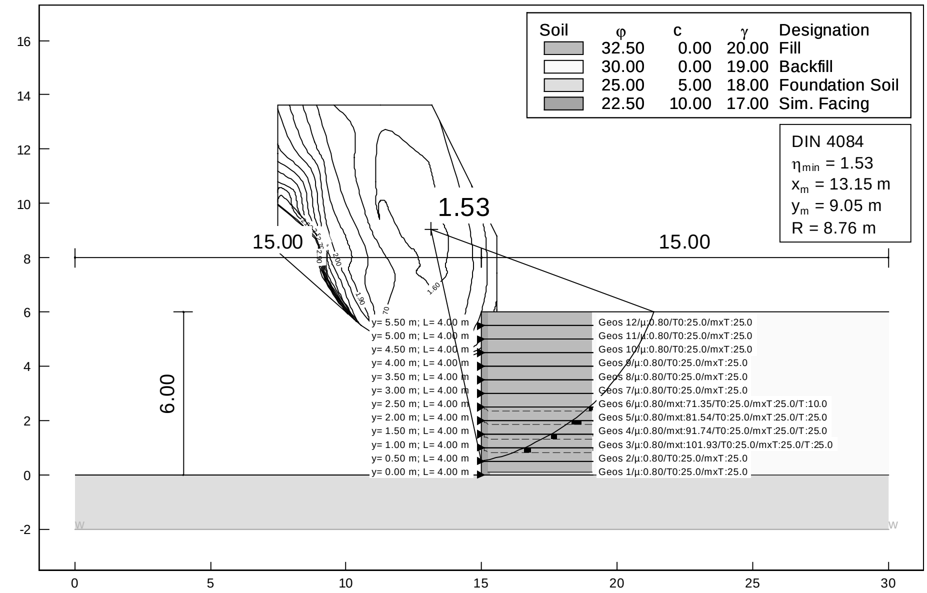


Figure 3b Reinforced vertical wall, long reinforcement, compound mode, Bishop

4.3 Third case: reinforced vertical wall with surcharge (Table 4)

Geometry and all other parameters are identical to those of the Second case, but additionally an uniformly distributed traffic surcharge was applied, because often it is a typical situation for vertical walls without backslope (horizontal surface). The surcharge varies from 10 kN/m² (a very modest value) to 30 kN/m² (for comparison: for German highways the value should be 33 kN/m²). The results are shown in Table 4 and in Figure 4.

Table ¨4 Third case analysis results (FOS):

| | SURCHARGE | INTERNAL MODE | | EXTERNAL MODE | | COMPOUND MODE | |
|---|---|---|---|---|---|---|---|
| | | L=2.5m[1)] | L=4.0m[1)] | L=2.5m[1)] | L=4.0m[1)] | L=2.5m[1)] | L=4.0m[1)] |
| BISH. | 10 kN/m$^2$ | 1.37 | 1.37 | 1.24 | 1.46 | 1.16 | 1.45 |
| | 20 kN/m$^2$ | 1.33 | 1.33 | 1.20 | 1.42 | 1.11 | 1.38 |
| | 30 kN/m$^2$ | 1.29 | 1.28 | 1.16 | 1.38 | 1.06 | 1.33 |
| JANB. | 10 kN/m$^2$ | 1.56 | 1.51 | 1.09 | 1.36 | 1.00 | 1.27 |
| | 20 kN/m$^2$ | 1.48 | 1.42 | 1.02 | 1.29 | 0.96 | 1.21 |
| | 30 kN/m$^2$ | 1.41 | 1.34 | 0.97 | 1.22 | 0.92 | 1.16 |

1) L: average reinforcement length

Some important issues should be pointed out. First, even for the 4m long geogrids (“balanced” design in Fig. 4) in many cases the FOS is not sufficient, although the minimum value is 1.15 > 1.00. Second, for the “cheap” design (short geogrids) in 8 cases (!) the FOS is less than the lowest FOS for the “balanced” design, and in 4 cases the FOS is even less than 1.0 (!). Third, for the short reinforcement the FOS-curves drop down more quickly with increasing surcharge; the difference between max FOS (= 1.57) and min FOS (= 0.92) amounts to 0.65 compared to (1.52 – 1.15) = 0.37 for the “balanced” solution with the longer geogrids. Thus, the “cheap” design (beside the FOS-insuffiencency) is much more sensitive to the load applied.

In the studied case, the ‘cheap’ wall has insufficient stability for surcharges p >10 kN/m$^2$ even in the most advantageous ‘internal’ mode (according to Bishop). For all other modes and methods its stability is insufficient even for p = 10 kN/m$^2$. According to Janbu compound mode analysis the ’cheap’ wall simply fails for any surcharge. Even considering the conservatism of Janbu this is alarming. The worst-case FOS for the “balanced” wall amounts to 1.15 for the (conservative) Janbu-analysis. The half of the cases for the ‘cheap’ wall have FOS below this one.

Checking only internal mode can have dramatic consequences. Not only the external mode, but also the compound mode have to be checked definitely, although they are not explicitly foreseen in the most popular codes incl. (BS 8006, 1995) or (EBGEO, 1997), and are not traditionally considered in projects in Brazil, especially the (very often critical) compound mode.

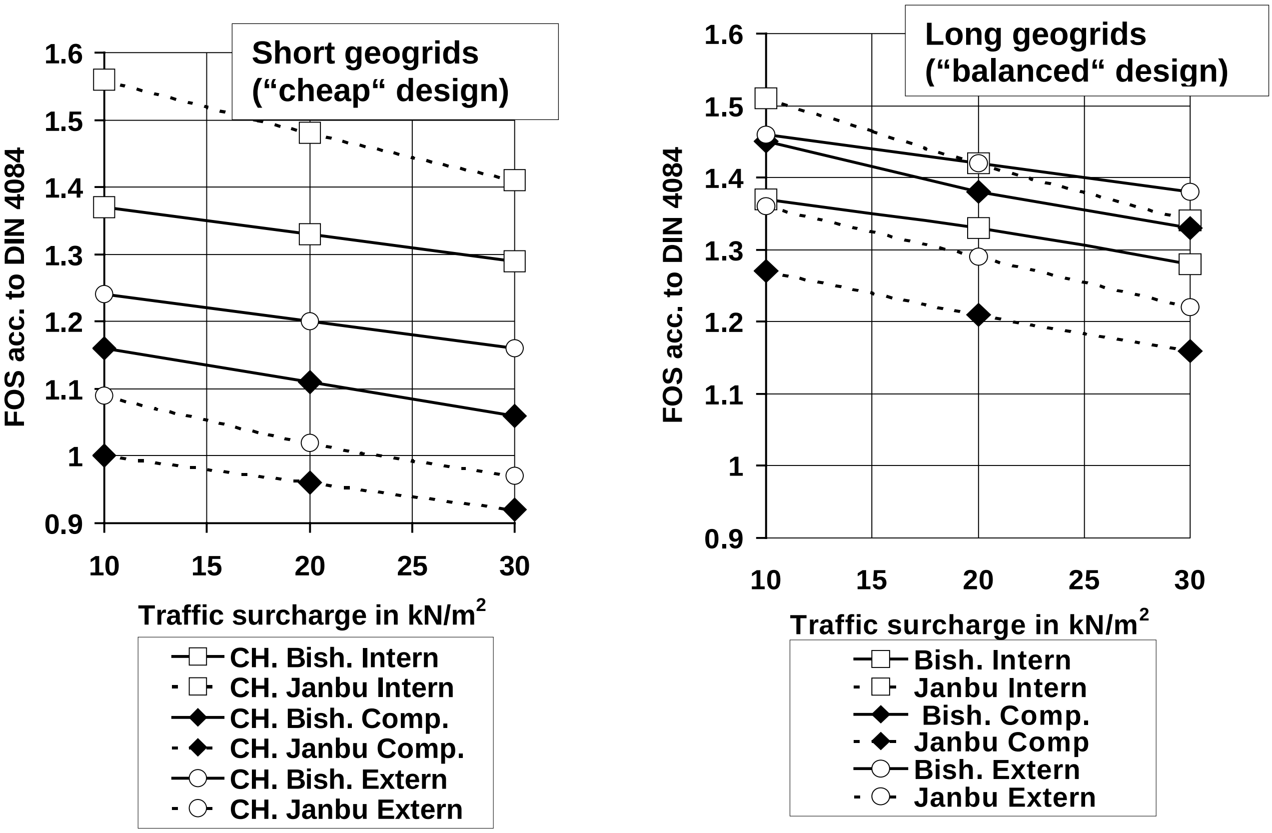


Figure 4 Graphs of the results of the Third case analysis

## 5 ADDITIONAL ANALYSES (Table 5)

For the typical First and Second cases additionally the influence of two parameters was shortly studied: the angle of internal friction of fill (φ) and the bond coefficient in the shear mode. The angle φ was assumed to be 45° instead of 32.5°. The bond coefficient (typically 1.0 in the shear-mode for Fortrac®-geogrids, see above) was assumed to be 0.7 (e.g. for other reinforcements). The background is that in the authors experience the fill strength is being often overestimated for performing “optimistic” designs, and for the same purpose the problem of bond coefficient in the shear (sliding) mode is being simply neglected.

Table 5 First and Second cases, compound mode, cheap typical section – results (FOS):

| | | ORIGINAL CONDITION (*) | HIGHER FILL SOIL RESISTANCE | LOWER INTERFACE RESISTANCE |
|---|---|---|---|---|
| FIRST CASE | BISHOP | 1.17 | 1.43 | - |
| | JANBU | 1.04 | 1.25 | 0.90 |
| SECOND CASE | BISHOP | 1.22 | 1.52 | - |
| | JANBU | 1.10 | 1.38 | 1.13 |

(*) Already presented in Tables 2 and 3.

It is obvious, that increasing φ results quickly in increased FOS (correct or not), and that lower shear-bond-coefficients could have dramatic consequences for the stability (e.g. FOS < 1.0).

In many cases, polygonal (Janbu) failure surfaces passing through the interface in the compound mode were the most critical. These results lead to some important conclusions:

- Bond capacity is of a great importance for a reinforcement material, and lack of this parameter can result in failure.
- Failure surfaces through the interfaces must always be tested for reinforcements with shear-bond-coefficient < 1.0 (Figure 5, detail).
- Compound mode must always be checked: it is the only possibility for checking critical interfaces (Figure 5).

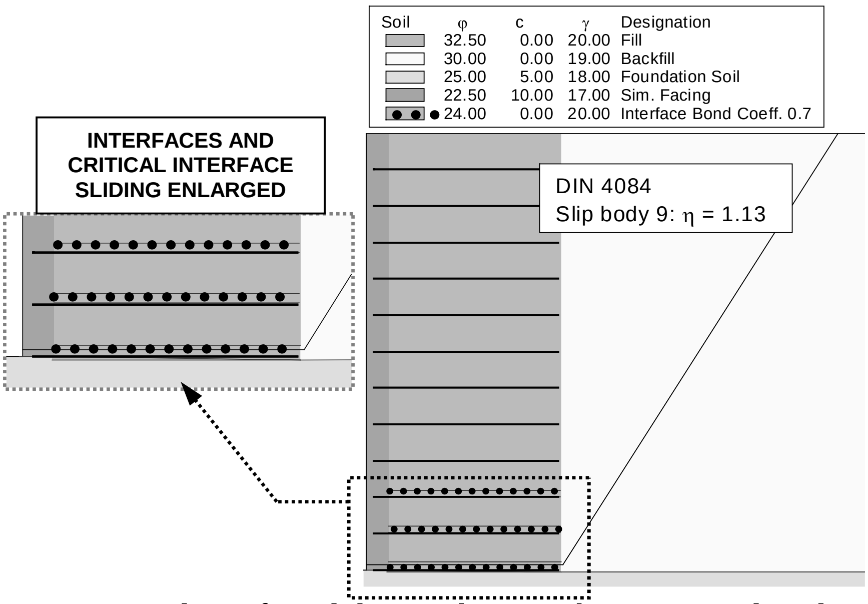


Figure 5 Typical interface sliding analysis; Janbu; compound mode

## 6 FINAL REMARKS

The so called “compound” mode of failure of reinforced slopes and walls has always to be checked, because very often it controls the design. Classic methods based on cylindrical (e.g. Bishop) and polygonal (e.g. Janbu) failure surfaces can be used. Neglecting the compound mode results in a “cheaper”, but risky solution. The results of the limit equilibrium analyses presented regarding this issue are confirmed by more sophisticated numerical FEM- (not shown) or FLAC-analyses (e.g. Leshchinsky & Vulova 2001).

Last note: In internal analysis an absurd may occur: for a configuration with longer reinforcement layers, lower FOS may be reached due to the larger area for searching for failure surfaces. It is definitely not realistic!

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